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Short Notes — Mathematics-I

BTEUP Diploma • 1st Semester (First Year — Common Curriculum, All 41 Branches) •

Short Notes

Session 2026-27

Short Revision Notes — prepared by KAMALDNP Academy from the official BTEUP 2026-27 syllabus. This is NOT a BTEUP board paper or publication.

These **short notes on Mathematics-I** collect the key points of every unit of the official BTEUP 2026-27 syllabus in one place. Each subtopic starts with the definition and then gives the formulas, rules and steps from which a complete **5-mark short answer** can be written in the examination.

Use the notes **with your syllabus PDF and the practice paper**: read one subtopic, close the notes and write the answer from memory, then check the formulas again. These are **revision notes only** — they revise the subject quickly before the exam and are not a replacement for your textbook or class notes.

Algebra & Trigonometry

Topics: Algebraic expressions and equations; trigonometric ratios, identities and heights and distances.

Algebraic expressions and equations

Definition: An algebraic expression is a combination of constants, variables and the operations of addition, subtraction, multiplication, division and powers, for example $3x^2 + 2x - 5$. An equation is a statement that two expressions are equal; its solution (root) is the value of the variable that satisfies it.

Identities to quote:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$). Roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The discriminant $D = b^2 - 4ac$ decides the nature of roots: D positive gives two real and unequal roots, $D = 0$ gives two real and equal roots, and D negative gives no real roots. Sum of roots $= -b/a$ and product of roots $= c/a$. A **linear equation** in one variable is solved by transposing terms; a pair of simultaneous linear equations is solved by elimination or substitution. Always verify a root by substituting it back into the original equation.

Trigonometric ratios and identities

Definition: For an acute angle θ in a right-angled triangle, the trigonometric ratios are ratios of pairs of sides: $\sin \theta = \text{perpendicular/hypotenuse}$, $\cos \theta = \text{base/hypotenuse}$, $\tan \theta = \text{perpendicular/base}$. The reciprocals are $\operatorname{cosec} \theta = 1/\sin \theta$, $\sec \theta = 1/\cos \theta$ and $\cot \theta = 1/\tan \theta$, and $\tan \theta = \sin \theta / \cos \theta$.

Standard values: at 0° , 30° , 45° , 60° , 90° the values of $\sin \theta$ are 0, $1/2$, $1/\sqrt{2}$, $\sqrt{3}/2$, 1; $\cos \theta$ takes the same values in reverse order; $\tan \theta$ is 0, $1/\sqrt{3}$, 1, $\sqrt{3}$ and undefined (∞) respectively.

Fundamental identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Complementary angles: $\sin(90^\circ - \theta) = \cos \theta$, $\cos(90^\circ - \theta) = \sin \theta$, $\tan(90^\circ - \theta) = \cot \theta$.

Compound and double angles:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = (\tan A + \tan B) / (1 - \tan A \tan B)$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

In an answer, state the identity first, then substitute values carefully with signs.

Heights and distances

Definition: Heights and distances problems find the height of a tower, building or pole, or the distance between two points, by measuring an angle from a known position and applying a trigonometric ratio — no direct measurement of the height is needed.

Two key angles: the **angle of elevation** is the angle between the horizontal line and the line of sight when the object is above eye level; the **angle of depression** is the same angle measured downwards when the object is below eye level. The angle of depression from the top of an object always equals the angle of elevation from its foot.

Working method:

- Draw a neat right-angled triangle and mark the known side and the required side.
- Choose the ratio that connects them: $\tan \theta = \text{height/distance}$ is used most often, because it joins the perpendicular and the base.
- Use $\sin \theta$ when the hypotenuse (for example a string, ladder or line of sight length) is known, and $\cos \theta$ when the hypotenuse and base are involved.
- Substitute the standard value of the ratio and solve.

Useful results: if the angle of elevation is 45° , height = distance; at 30° , height = distance/ $\sqrt{3}$; at 60° , height = $\sqrt{3} \times \text{distance}$. Always add the height of the observer where it is given.

Likely question: Prove that $\sin^2 \theta + \cos^2 \theta = 1$ and hence find $\sin \theta$ and $\tan \theta$ when $\cos \theta = 3/5$, or solve a heights-and-distances problem: from a point 30 m from the foot of a tower the angle of elevation of its top is 60° — find the height of the tower.

Calculus

Topics: Limits, differentiation and its applications; integration and its applications; definite integrals.

Limits

Definition: The limit of a function $f(x)$ as x approaches a is the value that $f(x)$ comes closer and closer to as x comes closer to a , written $\lim_{(x \rightarrow a)} f(x)$. The function need not be defined at $x = a$ itself; only the behaviour near a matters.

How a limit is evaluated:

- **Direct substitution** — put $x = a$; if a definite value comes, that is the limit.
- If substitution gives the indeterminate form $0/0$, first simplify: factorise and cancel, rationalise a surd numerator or denominator, or use a standard result.
- For $0/0$ forms, L'Hospital's rule may be used: differentiate the numerator and the denominator separately and then substitute.

Standard limits (learn with proof outline):

$$\lim_{(x \rightarrow 0)} \sin x / x = 1$$

$$\lim_{(x \rightarrow 0)} \tan x / x = 1$$

$$\lim_{(x \rightarrow 0)} (e^x - 1)/x = 1$$

$$\lim_{(x \rightarrow 0)} (a^x - 1)/x = \log_e a$$

$$\lim_{(x \rightarrow a)} (x^n - a^n)/(x - a) = n \cdot a^{n-1}$$

$$\lim_{(x \rightarrow \infty)} (1 + 1/x)^x = e$$

Algebra of limits: the limit of a sum, difference, product or quotient is the sum, difference, product or quotient of the limits, provided the denominator limit is not zero. A function is continuous at $x = a$ when $\lim_{(x \rightarrow a)} f(x) = f(a)$.

Differentiation and its applications

Definition: The derivative of $y = f(x)$ is the rate of change of y with respect to x : $dy/dx = \lim_{(h \rightarrow 0)} [f(x + h) - f(x)]/h$. Geometrically it is the slope of the tangent to the curve at that point.

Standard results:

$$d/dx (x^n) = n \cdot x^{n-1}$$

$$d/dx (\sin x) = \cos x; d/dx (\cos x) = -\sin x; d/dx (\tan x) = \sec^2 x$$

$$d/dx (e^x) = e^x; d/dx (a^x) = a^x \cdot \log_e a$$

$$d/dx (\log_e x) = 1/x; d/dx (\text{constant}) = 0$$

Rules: product rule $d(uv) = u \cdot dv + v \cdot du$; quotient rule $d(u/v) = (v \cdot du - u \cdot dv)/v^2$; chain rule for a function of a function, $dy/dx = (dy/dt) \cdot (dt/dx)$.

Applications:

- **Tangent and normal:** slope of tangent at (x_1, y_1) is $m = dy/dx$ there; tangent is $y - y_1 = m(x - x_1)$; the normal is perpendicular, with slope $-1/m$.
- **Maxima and minima:** at a turning point $dy/dx = 0$; it is a maximum if d^2y/dx^2 is negative there and a minimum if d^2y/dx^2 is positive.
- **Increasing and decreasing:** the function increases where dy/dx is positive and decreases where it is negative.
- **Rate of change and small errors:** for example dV/dt for a sphere uses $dV/dr \times dr/dt$; a small change is estimated by $\Delta y \approx (dy/dx) \cdot \Delta x$.

Integration and its applications

Definition: Integration is the reverse process of differentiation. If $d/dx F(x) = f(x)$, then $\int f(x) dx = F(x) + C$, where C is the constant of integration; without fixed limits it is called an indefinite integral.

Standard results:

$$\int x^n dx = x^{n+1}/(n + 1) + C \quad (n \neq -1)$$

$$\int (1/x) dx = \log_e |x| + C$$

$$\int e^x dx = e^x + C; \int a^x dx = a^x / \log_e a + C$$

$$\int \sin x dx = -\cos x + C; \int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

Methods:

- **Substitution:** put a part of the integrand, whose derivative is also present, equal to t .

- **By parts:** $\int u \, dv = u \cdot v - \int v \, du$; choose u by the ILATE order (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential).
- **Partial fractions:** split a proper rational fraction into simpler fractions and integrate each.

Applications: the area under a curve, the volume of a solid of revolution, and recovering a quantity from its rate — for example distance from velocity, velocity from acceleration. In every numerical, fix C from the given condition before writing the final answer.

Definite integrals

Definition: The definite integral of $f(x)$ between limits a and b is $\int(a \text{ to } b) f(x) \, dx = F(b) - F(a)$, where F is the indefinite integral of f . It gives a fixed number — the area between the curve, the x -axis and the ordinates $x = a$ and $x = b$ — and no constant C is written.

Properties (state one in a long answer):

$$\int(a \text{ to } b) f(x) \, dx = -\int(b \text{ to } a) f(x) \, dx$$

$$\int(a \text{ to } a) f(x) \, dx = 0$$

$$\int(a \text{ to } b) f(x) \, dx = \int(a \text{ to } c) f(x) \, dx + \int(c \text{ to } b) f(x) \, dx$$

$$\int(a \text{ to } b) f(x) \, dx = \int(a \text{ to } b) f(a + b - x) \, dx$$

$$\int(0 \text{ to } a) f(x) \, dx = \int(0 \text{ to } a) f(a - x) \, dx$$

Symmetric limits: for $\int(-a \text{ to } a) f(x) \, dx$, the value is $2\int(0 \text{ to } a) f(x) \, dx$ if f is an **even** function ($f(-x) = f(x)$), and zero if f is an **odd** function ($f(-x) = -f(x)$).

Area applications: area under $y = f(x)$ is $\int(a \text{ to } b) y \, dx$; area between two curves is $\int(a \text{ to } b) (\text{upper curve} - \text{lower curve}) \, dx$; volume of revolution about the x -axis is $V = \pi \int(a \text{ to } b) y^2 \, dx$. Use the modulus where the curve goes below the axis.

Likely question: Differentiate $x^3 \sin x$ using the product rule and find the equation of the tangent to $y = x^2$ at $(2, 4)$, or evaluate a definite integral such as $\int(0 \text{ to } \pi/2) \sin x \, dx$ using the properties of definite integrals.

Matrices & Determinants

Topics: Types of matrices; operations on matrices; determinants; solving systems of linear equations.

Types of matrices

Definition: A matrix is a rectangular arrangement of numbers in rows and columns, written inside brackets. A matrix with m rows and n columns has order $m \times n$ and contains $m \times n$ elements; the element in the i -th row and j -th column is written a_{ij} .

Main types:

- **Row matrix** — only one row; **column matrix** — only one column.
- **Square matrix** — rows = columns; its leading diagonal is the principal diagonal.

- **Null (zero) matrix** — every element is zero.
- **Diagonal matrix** — a square matrix whose off-diagonal elements are all zero; **scalar matrix** — a diagonal matrix whose diagonal elements are all equal.
- **Identity (unit) matrix I** — a scalar matrix with every diagonal element 1; it behaves like the number 1, since $AI = IA = A$.
- **Upper and lower triangular matrices** — square matrices with zeros below or above the principal diagonal respectively.
- **Symmetric matrix** — $A^T = A$; **skew-symmetric matrix** — $A^T = -A$, whose diagonal elements are necessarily zero.
- **Singular matrix** — a square matrix whose determinant is zero; a non-singular matrix has a non-zero determinant and alone possesses an inverse.

The **transpose** A^T is obtained by interchanging rows and columns. Two matrices are equal only when they have the same order and every corresponding element is equal.

Operations on matrices

Definition: The basic operations on matrices are addition, subtraction, multiplication by a scalar, matrix multiplication and transposition. Each operation has its own condition, and naming that condition is what earns full marks.

Addition and subtraction: possible only for matrices of the **same order**; add or subtract corresponding elements. Addition is commutative ($A + B = B + A$) and associative. **Scalar multiplication** kA multiplies every element of A by the number k .

Matrix multiplication: the product AB exists only when the number of columns of A equals the number of rows of B . If A is $m \times n$ and B is $n \times p$, then AB is $m \times p$, and each element of AB is the sum of products of a row of A with a column of B . Remember:

- In general $AB \neq BA$ — multiplication is **not commutative**, and one of AB , BA may exist while the other does not.
- It is associative: $(AB)C = A(BC)$, and distributive over addition.
- $(AB)^T = B^T A^T$ — the order reverses on transposing a product; $(A + B)^T = A^T + B^T$.
- $AI = IA = A$ for the identity matrix of matching order.

Inverse: for a non-singular square matrix, $A^{-1} = (\text{adj } A)/|A|$, and $AA^{-1} = A^{-1}A = I$. Show the conformability check before multiplying.

Determinants

Definition: A determinant is a single number associated with a **square** matrix. For a 2×2 matrix, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. A 3×3 determinant is evaluated by expanding along any row or column, using minors and cofactors with the checkerboard sign pattern $+$ $-$ $+$ / $-$ $+$ $-$ / $+$ $-$ $+$.

Properties (the heart of this topic):

- Interchanging two rows (or columns) changes the sign of the determinant.
- If two rows or columns are identical or proportional, the determinant is zero.
- Multiplying one row by k multiplies the determinant by k ; a common factor of a row may be taken outside.
- The determinant is unchanged if a multiple of one row is added to another row — this is the standard trick for creating zeros before expanding.
- The determinant of a triangular matrix is the product of its diagonal elements.
- $|A^T| = |A|$ and $|AB| = |A| \times |B|$.

Minors and cofactors: the minor of an element is the determinant left after deleting its row and column; the cofactor is the minor with its proper sign. The sum of the products of the elements of a row with their own cofactors gives the determinant — this is also how the adjoint matrix is built for finding an inverse.

Solving systems of linear equations

Definition: A system of linear equations, such as three equations in x , y and z , can be written in matrix form as $AX = B$, where A is the coefficient matrix, X the column of unknowns and B the column of constants. Solving it means finding values that satisfy all the equations together.

Matrix (inverse) method: if $|A| \neq 0$, then $X = A^{-1}B$. Compute $A^{-1} = (\text{adj } A)/|A|$, multiply it by B , and read off the unknowns. The condition $|A| \neq 0$ guarantees exactly one (unique) solution.

Cramer's rule: with D the determinant of the coefficient matrix, replace one column of D by the constants to get D_x , D_y (and D_z), then $x = D_x/D$, $y = D_y/D$, $z = D_z/D$ — valid only when $D \neq 0$.

What the determinant tells us:

- $D \neq 0$ — one unique solution; the system is consistent.
- $D = 0$ — no unique solution: the system is either inconsistent (no solution) or has infinitely many solutions.
- For homogeneous equations (all constants zero), $x = y = z = 0$ is always a solution; a non-zero solution exists only when $D = 0$.

In an examination answer, write the matrix form first, state the determinant's value, then apply the method — and verify the solution in one original equation.

Likely question: If A and B are given 2×2 matrices, find AB and BA and show that $AB \neq BA$, or solve the system $x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1$ by the matrix method or Cramer's rule.

Differential Equations

Topics: First-order differential equations and their simple applications.

First-order differential equations

Definition: A differential equation is an equation involving a dependent variable, an independent variable and the derivative of the dependent variable. Its **order** is the order of the highest derivative present and its **degree** is the power of that highest derivative (when the equation is a polynomial in the derivatives). A first-order equation contains dy/dx but no higher derivative.

Standard methods of solution:

- **Variables separable:** if $dy/dx = f(x) \cdot g(y)$, separate as $dy/g(y) = f(x) dx$ and integrate both sides. This is the first method to try.
- **Homogeneous equations:** if $dy/dx = F(y/x)$, substitute $y = vx$, so $dy/dx = v + x \cdot dv/dx$; the equation then becomes separable in v and x . Replace v by y/x at the end.
- **Linear equations:** for $dy/dx + P(x)y = Q(x)$, the integrating factor is $IF = e^{\int P dx}$, and the solution is $y \times IF = \int (Q \times IF) dx + C$. The same form applies with x treated as a function of y .
- **Bernoulli's equation** $dy/dx + Py = Qy^n$ is reduced to the linear form by dividing by y^n and substituting $z = y^{1-n}$.

Solution terms: the solution containing the arbitrary constant C is the **general solution**; the value of C found from a given condition gives the **particular solution**. Check a solution by differentiating it and substituting back.

Simple applications of first-order differential equations

Definition: Many practical quantities change at a rate proportional to the quantity itself or to some difference, and such statements translate directly into first-order differential equations. The skill tested is: form the equation from the verbal statement, solve it, and fix the constant from the given condition.

Growth and decay: if a quantity x grows at a rate proportional to x , then $dx/dt = kx$, giving $x = x_0 \cdot e^{(kt)}$ for growth (k positive) and $x = x_0 \cdot e^{(-kt)}$ for decay or depreciation (k negative). The same model covers population growth, bacterial growth, compound interest and radioactive decay; the half-life is found by putting $x = x_0/2$.

Newton's law of cooling: the rate at which a body cools is proportional to the difference between its temperature θ and the constant surrounding temperature θ_0 : $d\theta/dt = -k(\theta - \theta_0)$, whose solution is $\theta - \theta_0 = (\theta_1 - \theta_0)e^{(-kt)}$. Time-of-cooling questions are solved by using two temperature readings to eliminate k .

Simple electric circuit: for a series circuit with resistance R , inductance L and applied voltage E , Kirchhoff's law gives $L \cdot di/dt + Ri = E$ — a linear equation in the current i , solved with the integrating factor. In every application, finish by stating what the symbols mean and the unit of the answer.

Likely question: Solve the differential equation $dy/dx + y \cdot \tan x = \sec x$ (linear form) or $dy/dx = x/y$ by separating the variables, stating the order and degree of the equation first.

Quick Revision — One Line Each

- Quadratic roots: $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$; D positive, zero, negative means unequal real, equal real, no real roots.
- $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ — the three identities from which most proofs start.
- In heights and distances, $\tan \theta = \text{height/distance}$ is the most-used ratio; angle of depression equals angle of elevation.
- Standard limit: $\lim_{x \rightarrow 0} \sin x/x = 1$; a $0/0$ form is first factorised, rationalised or treated by a standard result.
- $d/dx (x^n) = n \cdot x^{n-1}$; product rule $u \cdot dv + v \cdot du$; at a maximum d^2y/dx^2 is negative, at a minimum it is positive.
- $\int x^n dx = x^{n+1}/(n+1) + C$; integration by parts: $\int u dv = u \cdot v - \int v du$, with u chosen by ILATE.
- Definite integral: $\int(a \text{ to } b) f(x) dx = F(b) - F(a)$; for symmetric limits an odd function gives zero and an even function gives twice the half-range value.
- Matrices can be added only in the same order, and multiplied only when columns of the first equal rows of the second; $AB \neq BA$ in general.
- Interchanging two rows of a determinant changes its sign; two identical rows make it zero; $|A| \neq 0$ means a unique solution and $A^{-1} = (\operatorname{adj} A)/|A|$.
- Variables separable: separate and integrate; linear $dy/dx + Py = Q$: IF = $e^{\int P dx}$ and $y \times$ IF = $\int(Q \times \text{IF}) dx + C$.
- Growth and decay both follow $x = x_0 \cdot e^{(kt)}$; Newton's cooling follows $d\theta/dt = -k(\theta - \theta_0)$.

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